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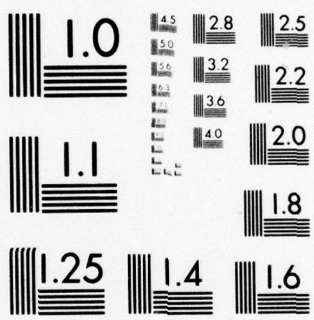

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GENERALIZED POISSON SHOCK MODELS

by

SHELDON M. ROSS

OPERATIONS RESEARCH CENTER

UNIVERSITY OF CALIFORNIA · BERKELEY

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APRIL 1978 ORC 78-6

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ABSTRACT

Suppose that shocks hit a device in accordance with a nonhomogeneous Poisson process with intensity function $\lambda(t)$ (lambda). The i^{th} shock causes a damage X_i. The X_i (sub i) are assumed to be independent and identically distributed positive random variables, and are also assumed independent of the counting process of shocks. Let $D(x_1, \ldots, x_n)$ (sub, sub) denote the total damage when n shocks having damages $x_1, \ldots, x_n$ (sub, sub) have occurred. It has previously been shown that the first time that $D(\underline{X})$ exceeds a critical threshold value is an increasing, failure rate average random variable whenever (i) $\lambda(t) = \lambda$ (lambda) and (ii) $D(\underline{x}) = \sum x_i$ (sum over x sub i). ~~We extend~~ this result is extended to the case where $\int_0^t \lambda(s)ds/t$ (integral from 0 to t of lambda) is nondecreasing in t and $D(\underline{x})$ is a symmetric, nondecreasing function. The extension is obtained by making use of a recent closure result for increasing failure rate average stochastic processes.

GENERALIZED POISSON SHOCK MODELS

by

Sheldon M. Ross

1. MODEL AND RESULT

We consider a unit subject to shocks which occur in accordance with a nonhomogeneous Poisson process with intensity function $\lambda(t)$, $t \geq 0$. We suppose that the i^{th} shock has a random damage X_i associated with it. The X_i, $i \geq 1$, are assumed to be independent positive random variables each having distribution F. They are also assumed to be independent of the counting process of shocks. We suppose that there is a function D such that if n shocks having values $x_1, \ldots, x_n$ have occurred by time t then

$$D(x_1, \ldots, x_n, \underline{0}) = \begin{cases} 1 & \text{if the unit has failed by } t \\ 0 & \text{otherwise.} \end{cases}$$

Letting T denote the time the unit fails, we have the following theorem.

Theorem 1:

If

(i) $T < \infty$ with probability 1,

(ii) $\int_0^t \lambda(s)ds/t$ is nondecreasing in t,

(iii) $D(x_1, \ldots, x_n, \underline{0}) = D\left(x_{i_1}, \ldots, x_{i_n}, \underline{0}\right)$ whenever $(i_1, \ldots, i_n)$ is a permutation of $1, 2, \ldots, n$,

(iv) $D(x_1, \ldots, x_i, \ldots) \leq D(y_1, \ldots, y_i, \ldots)$ whenever $0 \leq x_i \leq y_i$, $i \geq 1$, then T has an increasing failure rate average distribution.

Before proving the above theorem we need some preliminaries.

2. PRELIMINARIES

We start with some definitions.

Definitions:

(i) The nonnegative continuous random variable X having failure rate function $r(t) = \frac{d}{dt} P\{X \leq t\}/P\{X > t\}$ is said to have an increasing failure rate average distribution if $\int_0^t r(s)ds/t$ is nondecreasing in t.

(ii) The real valued stochastic process $\{X(t), t \geq 0\}$ is said to be an increasing failure rate average stochastic process if T_a has an increasing failure rate average distribution for all a, where $T_a = \inf\{t : X(t) > a\}$.

For an example of an increasing failure rate average stochastic process let $\{N(t), t \geq 0\}$ be a nonhomogeneous Poisson process with intensity function $\lambda(t)$ where $\int_0^t \lambda(s)ds/t$ is assumed to be nondecreasing in t. Further suppose that there is a value X_i associated with the i^{th} event. The X_i, $i \geq 1$, are assumed to be independent random variables each having the same distribution H, and they are also assumed to be independent of $\{N(t), t \geq 0\}$. Define $X(t)$ by

$$X(t) = \begin{cases} \max(X_1, \ldots, X_{N(t)}) & \text{if } N(t) \geq 1 \\ 0 \text{ if } N(t) = 0 . \end{cases}$$

Then it is easy to see that the failure rate function for $T_a = \inf\{t : X(t) > a\}$ is given by

$$r(t) = \lambda(t)(1 - H(a))$$

and so $X(t)$ is an increasing failure rate average process. We call it a "record process with value distribution H and intensity function $\lambda(t)$, $t \geq 0$."

The following theorem was proven by Ross in [2].

Theorem 2:

If $\{X_i(t) , t \geq 0\}$, $i = 1, \ldots, m$ are independent nondecreasing increasing failure rate average stochastic processes and if ϕ is a nondecreasing function then $\{\phi(X_1(t), \ldots, X_m(t)) , t \geq 0\}$ is also an increasing failure rate average process.

We are now ready for the

Proof of Theorem 1:

Let m be large and consider m independent record processes each having value distribution F and intensity function $\lambda(t)/m$ - call them $\{X_i(t)\}$, $i = 1, \ldots, m$. Now the shock model under consideration can be generated from these record processes by saying that a shock occurs whenever an event (from any of the m record processes) occurs and by letting its damage be the value associated with the Poisson event. Let N denote the number of shocks it takes until the component fails. Now if we define ϕ by

$$\phi(x_1, \ldots, x_m) = D(x_1, \ldots, x_m, \underline{0})$$

then it follows from Theorem 2 that the first time $D(x,(t), \ldots, x_m(t), \underline{0})$ equals 1 has an increasing failure rate average distribution. But as long as the first N shocks all come from different record processes this will be exactly the time the unit fails. Hence as the probability that all shocks until unit failure come from different record processes can be made arbitrarily close to 1 by letting m be large, the result follows by letting m go to infinity since the limit of increasing failure rate average random variables is also increasing failure rate average.||

<u>Remarks</u>:

(i) The special case where $\lambda(t) \equiv \lambda$ and

$$D(x_1, \ldots, x_n, \underline{0}) = \begin{cases} 1 & \text{if } \sum_1^n x_i > c \\ 0 & \text{otherwise} \end{cases}$$

was previously considered in [1].

(ii) It is easy to construct counterexamples to Theorem 1 if the symmetry condition on 1) is dropped.

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[1] Esary, J. D., A. W. Marshall and F. Proschan, "Shock Models and Wear Processes," _Annals of Probability_, Vol. 1, No. 4, pp. 627-650, (August 1973).

[2] Ross, S. M., "Multivalued State Component Systems," _Annals of Probability_, (to appear).